Delaunay triangulation is a powerful computational geometry technique used to partition a given set of points into a triangulated network. It is named after the French mathematician Boris Delaunay who first introduced this concept in 1934. Delaunay triangulation has numerous applications in various fields, including computer graphics, geographic information systems, and finite element analysis.

The Bowyer-Watson algorithm is one of the most commonly used algorithms for computing Delaunay triangulations. It is an incremental algorithm that adds points one by one to the triangulation. The algorithm starts with a super-triangle that contains all the given points. The super-triangle is a triangle that is larger than the bounding box that contains all the points. The algorithm then adds one point at a time to the triangulation and updates the triangulation accordingly.

When a new point is added to the triangulation, the algorithm first finds the triangle that contains the new point. This can be done by walking through the triangulation starting from any triangle and moving to adjacent triangles until a triangle containing the new point is found. Once the containing triangle is found, the algorithm removes the triangle and replaces it with three new triangles that are formed by connecting the new point to the three vertices of the containing triangle.

The Bowyer-Watson algorithm maintains the Delaunay property by flipping edges that violate the Delaunay criterion. An edge is said to be Delaunay if the circumcircle of the triangle formed by the edge and its adjacent triangles does not contain any other points in its interior. If an edge violates the Delaunay criterion, the algorithm flips the edge by removing the two triangles that share the edge and replacing them with two new triangles that are formed by connecting the non-shared vertices of the two triangles with the new point. The algorithm continues to add points and flip edges until all the points have been added and the Delaunay triangulation is complete.

One of the key features of Delaunay triangulation is the Voronoi diagram. The Voronoi diagram is a dual representation of the Delaunay triangulation. It is a partitioning of the plane into regions such that each region contains one point of the given set and all the points within a region are closer to that point than to any other point in the set. The Voronoi region for a point is defined as the set of all points that are closer to that point than to any other point in the set. The Voronoi diagram is a useful tool for analyzing spatial relationships between points and can be used for various applications, including computer graphics and image processing.

To construct the Voronoi diagram from the Delaunay triangulation, we first find the circumcenter of each triangle in the triangulation. The Voronoi vertex is then defined as the intersection of the perpendicular bisectors of the edges that form the triangle. The Voronoi region for a point is then defined as the region that contains all the points that are closer to that point than to any other point in the set.

In conclusion, Delaunay triangulation is a powerful computational geometry technique used to partition a set of points into a triangulated network. The Bowyer-Watson algorithm is a commonly used algorithm for computing Delaunay triangulations, and it maintains the Delaunay property by flipping edges that violate the Delaunay criterion. The Voronoi diagram is a dual representation of the Delaunay triangulation and is a useful tool for analyzing spatial relationships between points. The Voronoi region for a point is defined as the set of all points that are closer to that point than to any other point in the set.

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